TÉCNICO LISBOA

Quantum Dynamic Logic

João Miguel Neves Coelho

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Supervisor: Prof. Maria Cristina De Sales Viana Serôdio Sernadas

Examination Committee

Chairman: Prof. Paulo Alexandre Carreira Mateus
Supervisor: Prof. Maria Cristina De Sales Viana Serôdio Sernadas
Members of the Commitee: Prof. Jaime Arsénio de Brito Ramos
Prof. Maria Cristina De Sales Viana Serôdio Sernadas

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Resumo

A primeira menção a uma lógica baseada em Mecânica Quântica foi feita no artigo de Birkhoff e Neumann em 1936 (ver [3]). Aí, os conectivos da lógica quântica ($\sim$, $\wedge$, $\sqcup$) refletem as operações do reticulado de todos os subespaços fechados de um espaço de Hilbert. Como consequência, essa lógica não é uma extensão da lógica clássica. Nomeadamente, algumas propriedades da lógica clássica como a distributividade de $\wedge$ e $\sqcup$ já não são válidas. Atualmente existem muitas variantes da lógica quântica, algumas delas seguindo o paradigma de Birkhoff e von Neumann e outras onde as características quânticas são adicionadas à lógica clássica (para mais detalhes ver [4],[5],[6]). Nesta tese, iremos nos focar na lógica dinâmica quântica (ver [1],[2]) para programas quânticos, que é da segunda variante.
Lógica dinâmica quântica tem o mesmo papel para programas quânticos que a lógica dinâmica tem para programas clássicos. O papel desta última foi muito importante para a criação de técnicas de verificação para programação. Espera-se que a lógica quântica dinâmica faça o mesmo para programas quânticos, isto é, lidando com medições quânticas ,evoluções unitárias e entrelaçamento em sistemas quânticos compostos.

Palavras Chave

Lógica Quântica, Lógica Dinâmica, Mecânica Quântica, Computação Quântica

Abstract

The first logical account of quantum mechanics was presented by Birkhoff and Neumann's 1936 paper (see [3]). Therein, the connectives ($\sim$, $\wedge$, $\sqcup$) of quantum logic reflect the operations in the lattice of all closed subspaces of a Hilbert space. As a consequence this logic is not an extension of classical logic. Namely, some properties of classical logic like distributivity of $\wedge$ and $\sqcup$ are no longer valid. Nowadays there are many variants of quantum logic some of them following the paradigm of Birkhoff and von Neumann and others where quantum features are added to classical logic (for more details see [4],[5],[6]). In this thesis, we concentrate on quantum dynamic logic (see [1],[2]) for quantum programs which is from the latter variant.

Quantum dynamic logic has the same role for quantum programs as dynamic logic for classic programs. The role of the latter was very important for defining verification techniques for programming. It is expected that dynamic quantum logic would do the same to quantum programs, namely dealing with quantum measurements, unitary evolutions and entanglements in compound quantum systems.

Keywords

Quantum Logic, Dynamic Logic, Quantum Mechanics, Quantum Computation

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Chapter 1

Introduction

1.1 Motivation

There have been many news related with quantum computers, with titles such as “Game-Changing”, “Quantum Computing Race”, and “Quantum Supremacy”.

Quantum supremacy is just a scientific goal of having a quantum computer solve a problem that no classical computer is capable of solving in a feasible amount of time. This goal, however, does not specify the type of problem, it could be one that has no utility at all (solving a useful problem is still a long term goal).

Despite quantum supremacy not having intimidate application, many big companies (such as IBM, Google, Intel, Microsoft, and others) have spent millions of dollars into research on this area, as building a quantum computer is expensive and having one running are has great costs.

Since having a quantum computer working is expensive, it is vital that when a quantum program is run we have a guarantee that it works as intended. This is where Quantum Dynamic Logic fits in - it provides a way to show that a quantum algorithm is *sound* (*i.e.*, after running, it yields a result that is correct), just like Propositional Dynamic Logic does for classical algorithms.

1.2 Objective

Given the utility of Quantum Dynamic Logic, the goal of this thesis was to create a work that would be self-contained and a stepping-stone that would make Quantum Dynamic Logic easier to learn.

1.3 Challenges

When learning about Quantum Dynamic Logic there was four main challenges faced that this thesis tries to overcome.

Quantum Mechanics by its very nature is a hard[1] subject to learn therefore any area related to it will share this inherent difficulty. Besides Quantum Mechanics, Quantum Dynamic Logic requires or greatly benefits from knowledge about other areas (Quantum Logic, Propositional Dynamic Logic, Quantum

[1] There was a famous conference, the fifth Solvay conference where the most notorious physicists of the time gathered to discuss Quantum Mechanics, and seventeen were or ended being Nobel Prize winners.

Computation).
Thirdly, there was very little information regarding Quantum Dynamic Logic, searches yielded mostly the two main papers we already read for this thesis ([1],[2]). Lastly, said papers were time consuming to follow as they provide very few proofs of theirs statements.

1.4 What was Done

In order to reach the goal and overcome the challenges faced, introductory chapters were written about the required topics, as they were needed, proofs were done for the results that were not proven in the papers ([1],[2]), and, when needed, supplementary results and their proofs were added in order to ease the following of this work.

1.5 Topic Overview

To better help understand Quantum Dynamic Logic we can look at each of those three words individual. Logic is concerned with propositions and the laws to use such propositions that create a valid argument. Dynamic is related to change, so when we allow a system to change, as in a computer program, we need Dynamic Logic to reason about its properties. Finally, joining Quantum to Dynamic Logic, we have the same idea, we can reason about computer programs, but in this case we have quantum computers.

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